MIND MAP : LEARNING MADE SIMPLE

CHAPTER - 2

Inverse Trigonometric Functions

Domain and range of inverse trigonometric functions

(i) $y = \sin^{-1}x$. Domain $= [-1,1]$, Range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $y = \cos^{-1}x$. Domain $= [-1,1]$ Range $= [0, \pi]$

(iii) $y = \text{cosec}^{-1}x$. Domain $= R-(-1,1)$, Range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(iv) $y = \sec^{-1}x$. Domain $= R-(-1,1)$, Range $= [0, \pi] - \left\{\frac{\pi}{2}\right\}$

(v) $y = \tan^{-1}x$. Domain $= R$, Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(vi) $y = \cot^{-1}x$. Domain $= R$, Range $= (0, \pi)$.

Trigonometric functions

(I) $\sin : R \to [-1,1]$

(ii) $\cos : R \to [-1,1]$

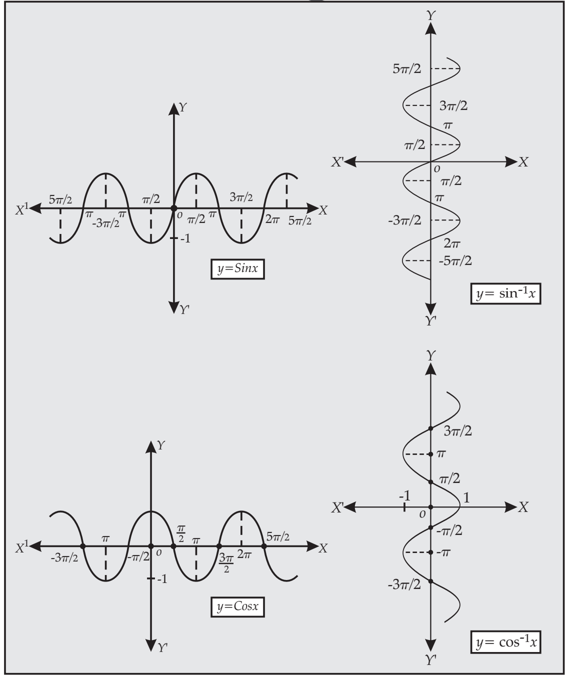
(iii) $\tan : R-\left\{x : x = (2n+1)\frac{\pi}{2} in \in Z\right\} \to R$

(iv) $\cot : R - \{x : x = h\pi, n \in Z\} \to R$

(v) $\sec : R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\} \to R-(-1,1)$

(vi) $\text{cosec} : R - \{x : x = h\pi, n \in Z\} \to R-(-1,1)$

Graphs of trigonometric functions and inverse trigonometric functions



Some important relations

(i) $y = \sin^{-1}x \Rightarrow x = \sin y$

(ii) $x = \sin y \Rightarrow y = \sin^{-1}x$

(iii) $\sin(\sin^{-1}x) = x$

(iv) $\sin^{-1}(\sin x) = x$

(v) $\sin^{-1}\frac{1}{x} = \text{cosec}^{-1}x$

(vi) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(vii) $\cos^{-1}\frac{1}{x} = \sec^{-1}x$

(viii) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

(ix) $\tan^{-1}\frac{1}{x} = \cot^{-1}x$

(x) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

(xi) $\sin^{-1}(-x) = -\sin^{-1}x$

(xii) $\tan^{-1}(-x) = -\tan^{-1}x$

(xiii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(xiv) $\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$

(xv) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

(xvi) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

(xvii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$

(xviii) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1+x^2}{1-x^2}$

For *eg* : to find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y$,

$\Rightarrow \sin y = \frac{1}{\sqrt{2}}$ The range or the principal value branch of $\sin^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

and $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ So, the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$

$\sin^{-1}x \neq (\sin x)^{-1}.(\sin x)^{-1} = \frac{1}{\sin x}$ and same for other trigonometric functions.

Principal value branch and principal value

The range of an inverse trigonometric function is the principal value branch and those values which lies in the principal value branch is called the principal value of that inverse trigonometric functions.